

Boson stars in SU(2) Yang-Mills-scalar field theories

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Abstract

We present new spherically symmetric solutions of an SU(2) Einstein-Yang-Mills model coupled to a doublet of scalar fields. Sequences of asymptotically flat, Yang-Mills-boson star-type configurations are constructed numerically by considering an appropriate time-dependent ansatz for the complex scalar field and a static, purely magnetic SU(2)-Yang-Mills potential. Both nodeless as well as solutions with nodes of the scalar field and gauge potential are considered. We find that these solutions share many features with the “pure” boson stars.

1 Introduction

Although there is still no direct evidence for the existence of scalar fields, there are many theoretical reasons that these fields might play an important role in the evolution and the structure of the Universe. It is possible that a fraction of the bosonic dark matter has collapsed to form stellar type objects - boson stars. Boson stars (BS) are localised, static configurations of gravitationally bound zero temperature scalar particles, the complex scalar field possessing a harmonic time dependence. The study of BS started with the work of Kaup [1] and Ruffini and Bonazzalo [2], who found asymptotically flat, spherically symmetric equilibrium solutions of the Einstein-Klein-Gordon equations. These configurations are “macroscopic quantum states” and are only prevented from collapsing gravitationally by the Heisenberg uncertainty principle.

The BS share many features with their fermionic counterparts, presenting however many interesting differences. For example, BS also exhibit a critical mass and critical particle number. Later work considered the self-interacting case [3, 4, 5] or a non-minimal coupling of the scalar field to gravity [6]. Boson stars in the presence of a dilaton or an axidilaton have also been studied by various authors [7], as well as boson-fermion stars [8].

All these models have demonstrated the same characteristic: new interactions tend to increase the critical values of mass and particle number, although the particular values are very model dependent. The stability against perturbations around the equilibrium state has been discussed also by a number of authors [9]-[12]. An extensive review of the boson star properties is given in [13], [14] and more recently in [15].

Jetzer and van der Bij extended the BS model to include the coupling with a $U(1)$ gauge group [16]. Here, the scalar field regularizes the central singularity of the Reissner-Nordström solution, which otherwise would

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necessarily be present in the pure Einstein-Maxwell theory. A natural generalization of these charged BS configurations is to consider a larger gauge group. In this case, there are regular configurations even in the absence of a scalar field, as proven by the Bartnik-McKinnon (BM) static, time-independent solution of the coupled SU(2) Einstein-Yang-Mills (EYM) equations [17] (see [18] for a general discussion of the properties of this type of solutions).

In this paper, we extend the analysis of [16] to include a complex doublet of scalar fields coupled to an SU(2) non-abelian gauge field ¹. In the static case, and for a symmetry breaking scalar potential, this system admits sphaleron solutions, describing the top of the potential-energy barrier separating gauge-inequivalent classical vacua [19]. The BM configurations are recovered in the limit of vanishing scalar field [22].

Here we present both numerical and analytical arguments for the existence of a new type of solution of the coupled EYM-scalar field equations, which combines the basic properties of both BS and BM models. For any value of the scalar field at the origin, the BS solutions can be generalized to include a BM particle inside. Different from the sphaleron case, however, the magnitude of the scalar field is zero at infinity.

The paper is structured as follows: in the next Section we present the general framework and analyse the field equations and boundary conditions. In Section 3 we present our numerical results. We conclude with Section 4, where our results are summarized.

2 General framework and equations of motion

2.1 Basic ansatz

Our study of the EYM-scalar field system is based upon the action

$$S = \int d^4x \sqrt{-g_m} \left[\frac{1}{16\pi G} \mathcal{R} - (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi) - \frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} \right], \quad (1)$$

where, following the standard model, we take the scalar field Φ to be a complex doublet. G is the gravitational constant and g_m is the determinant of the metric tensor.

Here D_μ is the usual gauge-covariant derivative

$$D_\mu \Phi = \partial_\mu \Phi + g(A_\mu^a \sigma_a) \Phi, \quad (2)$$

while

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g\epsilon_{abc} A_\mu^b A_\nu^c \quad (3)$$

is the SU(2) field strength tensor and g is the gauge coupling constant.

In this letter we will restrict our analyse to the case $V(\Phi) = \mu^2 \Phi^\dagger \Phi$, where μ is the scalar field mass, without including a scalar self-interaction term. As found in [4], although the inclusion of a $\lambda|\Phi|^4$ term drastically changes the value of the maximum mass and the corresponding critical central density of the boson star solutions, the qualitative features are essentially similar to the non-self interaction case.

The action is invariant under a global phase rotation $\Phi \rightarrow \Phi e^{-i\alpha}$ which implies the existence of a conserved current

$$J^\mu = ig^{\mu\nu} \left((D_\nu \Phi^\dagger) \Phi - (D_\nu \Phi) \Phi^\dagger \right). \quad (4)$$

This gives an associated conserved charge, namely, the number of scalar particles:

$$N = \int d^3x \sqrt{-g} J^t. \quad (5)$$

Since we assume spherical symmetry, it is convenient to use the metric in Schwarzschild-like coordinates:

$$ds^2 = \frac{dr^2}{B(r)} + r^2(d\theta^2 + \sin^2\theta d\varphi^2) - \sigma^2(r)B(r)dt^2, \quad (6)$$

¹Asymptotically anti de-Sitter q-stars interacting with a nonabelian field have been studied recently in [23]. However, the properties of these notopological solitons are rather different as compared with the solutions discussed in this paper.

where $B(r) = 1 - 2m(r)/r$. $m(r)$ may be interpreted as the total mass-energy within the radius r ; its asymptotic value gives the total ADM mass of the solutions.

Similar to the pure BS case, we can define a configuration radius

$$R = \frac{1}{N} \int d^3x \sqrt{-gr} J^t . \quad (7)$$

We consider only static gauge fields, and for simplicity we assume $A_t^a = 0$. The most general spherically symmetric SU(2) ansatz is parametrized by

$$A_i^a = \frac{1 - W(r)}{gr} \epsilon_{aij} \hat{x}_j + \frac{\tilde{W}(r)}{gr} (\delta_{ia} - \hat{x}_i \hat{x}_a) + \frac{a(r)}{gr} \hat{x}_i \hat{x}_a , \quad (8)$$

while for the scalar field we choose

$$\Phi = e^{-i\tilde{\omega}t} [\phi(r) + iK(r)(\hat{x}^a \sigma_a)] \begin{pmatrix} 0 \\ 1 \end{pmatrix} . \quad (9)$$

It is well known that the Ansatz for the matter fields is plagued with a residual gauge symmetry. Along with [24], we fix the gauge by imposing the axial gauge

$$x^i A_i = 0 \quad \Rightarrow \quad a(r) = 0 . \quad (10)$$

In this paper we consider the consistent truncation of the full model $\tilde{W}(r) = K(r) = 0$, which corresponds to imposing the symmetry of the fields under the parity operator. Note that the original sphaleron solution of Klinkhamer and Manton [19] as well as the gravitating generalizations [22] have been constructed in this reduced ansatz. However, solutions with $\tilde{W}(r) \neq 0$, $K(r) \neq 0$ (so-called ‘‘bi-sphalerons’’) have been constructed in [20, 21]. The general spherically symmetric configurations have also been used in [24].

2.2 Field equations and boundary conditions

Within this ansatz, the classical equations of motion can be derived from the following 2-dimensional action:

$$S = \int dt dr \sigma \left[\frac{m'}{4\pi G} - \frac{1}{g^2} \left(BW'^2 + \frac{(W^2 - 1)^2}{2r^2} \right) - \left(Br^2 \phi'^2 + \frac{1}{2} \phi^2 (W - 1)^2 + \mu^2 r^2 \phi^2 - \frac{\tilde{\omega}^2 \phi^2 r^2}{B\sigma^2} \right) \right] , \quad (11)$$

where the prime denotes the derivative with respect to r .

The field equations reduce to the following system of four non-linear differential equations:

$$\begin{aligned} m' &= 4\pi G \left(\frac{1}{g^2} \left(BW'^2 + \frac{(W^2 - 1)^2}{2r^2} \right) + Br^2 \phi'^2 + \frac{1}{2} \phi^2 (W - 1)^2 + \mu^2 r^2 \phi^2 + \frac{\phi^2 r^2 \tilde{\omega}^2}{B\sigma^2} \right) , \\ \sigma' &= \frac{8\pi G \sigma}{r} \left(\frac{W'^2}{g^2} + r^2 \phi'^2 + \frac{\phi^2 r^2 \tilde{\omega}^2}{\sigma^2 B^2} \right) , \\ (\sigma BW')' &= \frac{\sigma W}{r^2} (W^2 - 1) + \frac{1}{2} g^2 \sigma \phi^2 (W - 1) , \\ (\sigma Br^2 \phi')' &= \frac{\sigma \phi}{2} (W - 1)^2 + \mu^2 \sigma r^2 \phi - \frac{\phi r^2 \tilde{\omega}^2}{\sigma B} . \end{aligned} \quad (12)$$

The regularity of the solution at the origin, the finiteness of the ADM mass and the requirement that the metric (6) approaches the Minkowski metric for $r \rightarrow \infty$ lead to definite boundary conditions for the four functions m , σ , W , ϕ . As far as the metric functions are concerned we have to impose

$$m(0) = 0 \quad , \quad \sigma(\infty) = 1 . \quad (13)$$

The remaining functions have to obey

$$W(0) = 1, \quad \phi'(0) = 0, \quad W(\infty) = (-1)^p, \quad \phi(\infty) = 0 , \quad (14)$$

(where p is a positive integer), which are the usual boundary conditions for the BS and BM configurations, respectively. Thus, for finite energy solutions, the term $\tilde{\omega}^2\phi^2$ in (11) forces ϕ to tend to zero at infinity, which excludes a “boson star-sphaleron” hybrid model with a scalar field potential.

The field equations imply the following behaviour for $r \rightarrow 0$ in terms of three parameters (b, ϕ_0, σ_0) :

$$\begin{aligned}\phi(r) &= \phi_0 + \frac{\phi_0}{6}(\mu^2 - \tilde{\omega}^2\sigma_0^2)r^2 + O(r^3), & W(r) &= 1 - br^2 + O(r^4), \\ m(r) &= \frac{1}{12\pi Gg^2}(6b^2 + \phi_0^2g^2(\mu^2 - 3\tilde{\omega}^2\sigma_0^2))r^3 + O(r^4), & \sigma(r) &= \sigma_0 + \left(\frac{b^2}{\pi Gg^2} + \frac{\phi_0^2\tilde{\omega}^2\sigma_0^3}{4\pi G}\right)r^2 + O(r^3).\end{aligned}\tag{15}$$

A similar analysis for $r \rightarrow \infty$ reveals that the leading order asymptotic behaviour of the metric functions is determined by the gauge field only, while the scalar field asymptotic form is similar to the pure BS case:

$$\begin{aligned}\phi(r) &= \phi_1 r^\Delta e^{-\sqrt{\mu^2 - \tilde{\omega}^2}r} + \dots, & \text{with } \Delta &\equiv -1 + M \frac{2\tilde{\omega}^2 - \mu^2}{\sqrt{\mu^2 - \tilde{\omega}^2}}, \\ W(r) &= (-1)^p - \frac{w_1}{r} + \dots, & m(r) &= M - \frac{4\pi G}{g^2} \frac{w_1^2}{r^3} + \dots, & \sigma &= 1 - \frac{2\pi G}{g^2} \frac{w_1^2}{2r^4} + \dots.\end{aligned}\tag{16}$$

To perform numerical computations and order-of-magnitude estimations, it is useful to have a new set of dimensionless variables. This is obtained by using the following rescaling

$$r \rightarrow r\sqrt{4\pi G}/g, \quad \phi(r) \rightarrow \phi/\sqrt{8\pi G}, \quad m(r) \rightarrow m(r)\sqrt{4\pi G}/g.\tag{17}$$

The system of differential equations (12) with the above boundary conditions then depends on only two coupling constant, namely:

$$\alpha = \mu \frac{\sqrt{4\pi G}}{g} = \frac{\sqrt{4\pi}}{g} \frac{\mu}{M_{Pl}}, \quad \omega = \tilde{\omega} \frac{\sqrt{4\pi G}}{g} = \frac{\sqrt{4\pi}}{g} \frac{\tilde{\omega}}{M_{Pl}}\tag{18}$$

where $M_{Pl} = \sqrt{G}$ is the Planck mass scale.

Note that the factor ω could be absorbed into the definition of the metric function σ and this would change the boundary conditions for σ , with $\sigma(\infty) \neq 1$. The system of differential equations (12) would then depend only on α .

We have however kept two coupling constants, α and ω . In our numerical computations, we have used α and $\phi(0)$ as free parameters and obtained ω as function of $\phi(0)$.

3 Numerical results

The equations of motion (12) have been solved for a range of values of the scalar field at the origin, $\phi(0)$, and several values of α . As expected, these solutions have many features in common with the pure BS and BM solutions; they also present new features that we will point out in the discussion.

For each choice of ϕ_0 , the field equations have a solution with the right asymptotics for $r \rightarrow \infty$ only when $(\sigma(0), W''(0) \sim b)$ in the expansion at the origin (15) take on certain values. The value of the shooting parameter b decreases with increasing ϕ_0 . For the same value of ϕ_0 , different values of σ_0 corresponds to different numbers of nodes of the scalar field.

In the following, we denote by p the number of nodes of the gauge field functions and by n the number of nodes of the scalar field function. For the considered configurations we could not find non-trivial nodeless gauge field function. Thus $p = 0$ implies $W(r) \equiv 1$.

3.1 Known configurations: Bartnik-McKinnon and “pure boson” star solutions

If we set the scalar field to zero, the corresponding equation is trivially satisfied and we are left with the spherically symmetric Einstein-Yang-Mills system, whose solutions are the BM configurations [17]. They consist of a discrete family of smooth solutions uniquely characterized by the number of nodes p of the function $W(r)$, with $p \geq 1$. For these solutions the metric function $B(r)$ possesses a minimum at some finite value of r .

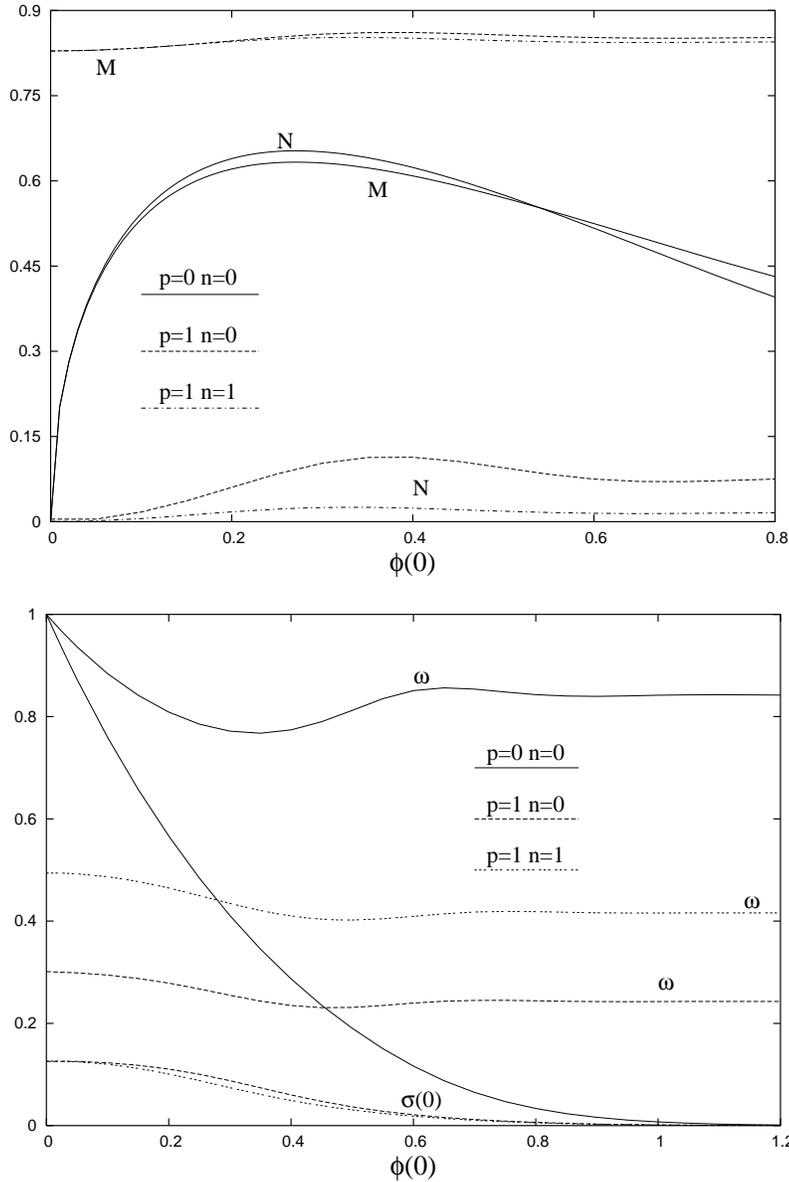


Figure 1. The mass-parameter M , the particle number N , the value of the metric function $\sigma(r)$ at the origin, $\sigma(0)$, and the parameter ω are represented as a function of the value of the scalar field at the origin $\phi(0)$ for “pure” boson stars ($p = 0, n = 0$) and $p = 1, n = 0, 1$ Yang-Mills-boson star configurations.

The metric function $\sigma(r)$ increases monotonically from $r = 0$ (with $0 < \sigma(0) < 1$) to $r = \infty$ with $\sigma(\infty) = 1$. Boson-star solutions discussed in this paper can be regarded as deformations (i.e. with $\phi(r) \neq 0$) of each solution of the BM sequence.

In the case when the Yang-Mills field is set to its vacuum value the parameter α can further be rescaled in the radial variable, and the equations reduce to those of the “pure” boson-star. The scalar field can be gradually deformed by setting $\phi(0) = \phi_0 > 0$ and the corresponding solution can be constructed numerically [1, 2]. The configurations can be characterized namely by their mass ($m(\infty)$) the values $\phi(0)$, $\sigma(0)$ and B_m (i.e. the minimal value of the metric function $B(r)$) and by the value ω .

The behaviour of M and N as a function of $\phi(0)$ is well known in this case; the mass and particle number rise with increasing $\phi(0)$ to the maximum values $M_{max} = 0.633$ and $N_{max} = 0.653$ (see Figure 1). Then M and N decrease, oscillate a bit and approach asymptotically a constant value.

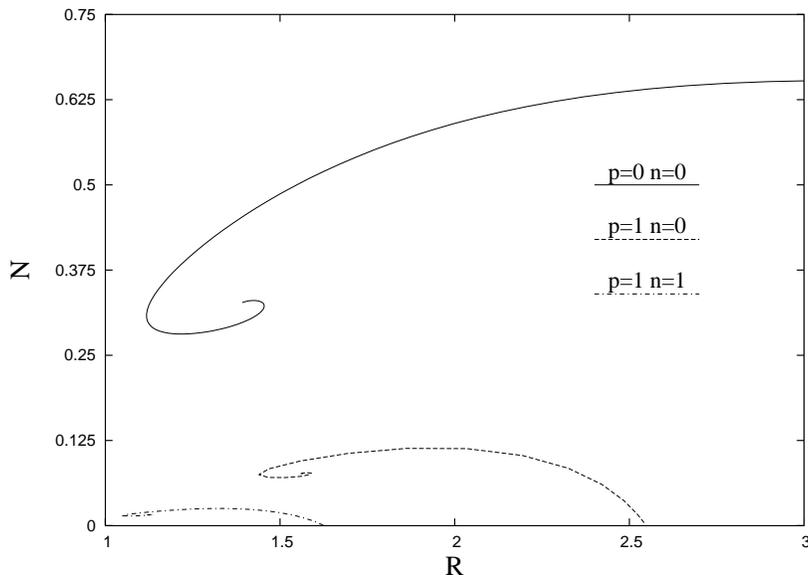


Figure 2. The particle number N is represented as a function of the effective radius R for the same configurations as in Figure 1.

When the value of the scalar field at the origin $\phi(0)$ becomes large, the value $\sigma(0)$ decreases and tends asymptotically to zero. However, a new phenomenon appears in this limit. Namely the metric function $B(r)$ develops a second minimum between $r = 0$ and the global minimum. This will be discussed in more detail in the context of YM-boson stars.

Figure 1 also illustrates the fact that BS solutions exist only on a definite interval of ω and that on some subinterval ($0.76 \geq \omega \geq 0.85$) several solutions with the same ω can exist. Here we have examined only the case when the scalar field possesses no nodes (i.e. $n = 0$), but solutions exist for which the scalar field function possesses one or more zeros, i.e. $n \neq 0$.

3.2 Yang-Mills-Boson-star: $p = 1, 2, 3$

We have constructed (again by imposing a fixed value for $\phi(0) = \phi_0 > 0$) families of solutions with $n = 0, 1, 2$, respectively 3 nodes of the scalar field function and $p = 1, 2, 3$ nodes of the gauge field function for varying α . We first discuss the solution for a generic value of the mass parameter α , namely $\alpha = 1$. The results turn out to be qualitatively the same for other values of this parameter. A discussion of the domain of existence of solution in the parameter α is presented at the end of this section.

The data characterizing the $n = 0$ and the $n = 1$ solutions is shown in Figures 1, 2. For the case $n = 2$ the pattern remains qualitatively the same. These graphs present the same quantities as in the case of the “pure” boson stars.

The main difference is that the p -th BM solution is approached in the limit $\phi(0) \rightarrow 0$. Accordingly, the ADM-mass does not vanish in this limit, but is equal to that of the corresponding BM solution. The particle number behaves like $\phi(0)^2$ and as a consequence the star radius stays finite. Excluding this latter point, note that the behaviour of M and N as a function of $\phi(0)$ is similar to that of the “pure” boson stars.

We see also that the mass and the parameter ω depend only weakly on $\phi(0)$ (this holds true also for B_m which is not shown in the Figure), while $\sigma(0)$ decreases monotonically for increasing $\phi(0)$. As in the case without a non-abelian field, we see that the particle number and the mass have their extrema, in particular their maximum, at the same value of the central density (characterized by $\phi(0)$), decreasing monotonically to a constant asymptotic value. The behaviour of the particle number (or mass) in dependence on the radius is similar to the “pure” BS case (see Figure 2). In both cases the particle number versus radius curve shows a

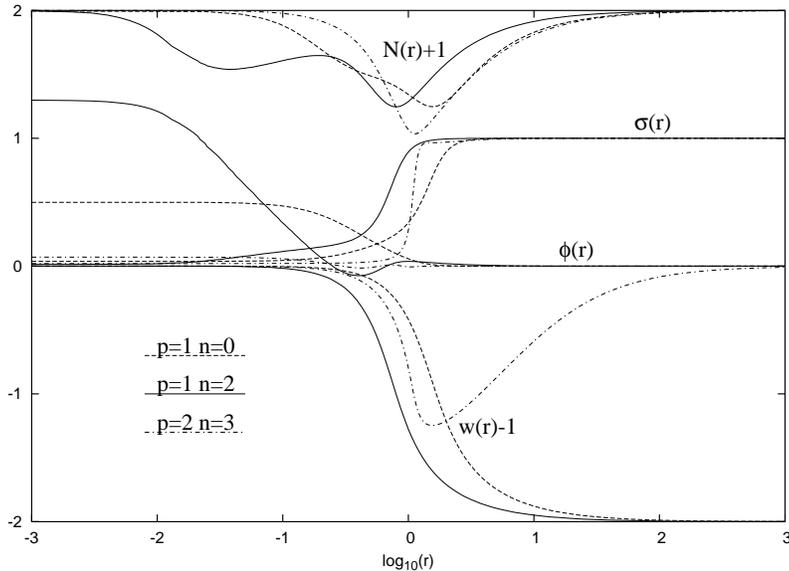


Figure 3. The profiles of typical Yang-Mills-boson stars with $\alpha = 1$ are plotted for several values of p, n .

counterclockwise inspiralling at the critical points corresponding to a maximum of N . Another point which is worth pointing out is that the various solutions exist only on definite intervals of the parameter ω and that ω is not a monotonic function of $\phi(0)$. This parameter converges to a finite value (depending on n, p) for $\phi(0) \rightarrow 0$. Note that the values of ω for a fixed value of $\phi(0)$ are smaller in the case of Yang-Mills boson stars as compared to the “pure” boson stars and for the Yang-Mills boson stars themselves smaller for $n = 0$ as compared to $n = 1$.

Another feature which the Yang-Mills boson star shares with the “pure” boson star is the fact that, when the scalar field’s value at the origin becomes large enough, the metric function $B(r)$ develops a second local minimum, situated inside the global minimum. This is illustrated in Figure 3, where the profiles of the functions $B(r), \sigma(r), W(r), \phi(r)$ are shown for three different combinations of p and n . Note that for $p = 1$ and $n = 2$ the mentioned second minimum, which we associate with a shell-like structure of the solutions, appears.

We remark here that our numerical analysis was stopped due to numerical difficulties related to the smallness of $\phi(0)$ and we can thus not make any conclusive statement about whether the second minimum becomes the global minimum if $\phi(0)$ is small enough.

We have also managed to construct solutions with two nodes in the gauge field, i.e. $p = 2$ and n nodes in the scalar field. The profiles of the functions $N(r), \sigma(r), W(r)$ and $\phi(r)$ for the $p = 2$ and $n = 3$ solution are shown in Figure 3. The qualitative behaviour of N and M is the same as for $p = 1$. The most sensitive parameter to the number of nodes p is the frequency ω . Choosing $\phi(0)$ rather small (e.g. $\phi(0) = 0.1$) and concentrating on the $n = 0$ solution, we observe that $\omega \sim 0.3$ for $p = 1$ and $\omega \sim 0.9$ for $p = 2$. This has important consequence on the domain of existence of solutions in the α - $\phi(0)$ -plane.

Let us finally discuss this domain of existence. The precise determination of this domain is a huge task, which we don’t aim at in this letter, however, we can make some statements about the qualitative behaviour. We first assume that $\phi(0)$ is fixed (say $0 < \phi(0) < 1.5$) and α is varied. Our numerical analysis suggests that the parameter ω decreases considerably when α decreases. A consequence of this is that the quantity $\sqrt{\alpha^2 - \omega^2}$, which appears in the asymptotic analysis (see (16)) becomes imaginary for $\alpha < \alpha_{cr}$ such that exponentially localized solutions cease to exist for such values. For $p = 1$, we find numerically $\alpha_{cr} \approx 0.16$ for $\phi(0) = 0.1$ and $\alpha_{cr} \approx 0.12$ for $\phi(0) = 0.5$. For $p = 2$ the critical value of α turns out to be larger, e.g. $\alpha_{cr} \sim 0.85$ for $\phi(0) = 0.1$.

We could hardly find any relevant critical phenomenon limiting the solution pattern when increasing α . For instance we checked that both the metric and the scalar field stay perfectly regular up to $\alpha \sim 10.0$.

For fixed α and increasing $\phi(0)$ we have found that the value of the metric function σ at the origin, $\sigma(0)$, becomes very small for large $\phi(0)$, while the minimum of the function $B(r)$ stays finite. This suggests that the solution becomes singular at the origin for large enough $\phi(0)$. However, because the numerical calculations become very difficult for large $\phi(0)$, we refrain from making any further conclusions here.

4 Conclusions and further remarks

In this paper we have investigated a new type of configuration combining the basic properties of two well known solutions: boson stars for a time-dependent complex scalar field coupled to gravity on the one hand and Bartnik-McKinnon particle-like solutions of the $SU(2)$ Einstein-Yang-Mills system on the other hand.

The properties of these Yang-Mills boson star solutions differ considerably from their abelian counterparts discussed in [16]. Note also that these solutions represent the simplest examples of Yang-Mills boson star configurations. We expect that static, respectively rotating axially symmetric solutions as well as dilaton generalizations may exist as well.

The Einstein-Yang-Mills system possesses also black hole solutions, so-called “coloured” black holes, which are counterexamples to the no-hair conjecture [18]. There are no black hole analogues to the “pure” BS configurations [25] though. However, the introduction of an $SU(2)$ gauge field invalidates the arguments presented in [25]. The existence of black hole counterparts of the configurations discussed in this paper is thus an open question.

An important physical question when discussing self-gravitating configurations is whether these solutions are stable. The stability of the Yang-Mills boson stars can be studied in general by considering a perturbation about the classical solutions and solving the linearized equations. This leads to a system of coupled Schrödinger (or Sturm-Liouville) equations, whose analysis is a complicated task. However several useful informations about the negative modes of the solutions can be obtained by arguments based of catastrophe theory and the inspection of bifurcations and/or cusps occurring in the pattern of the solutions [26, 12].

It is known that the p -node BM solution is plagued with $2p$ unstable modes [27], in other words it is a sphaleron for $p > 0$. As a consequence it is very likely that, at least for small values of $\phi(0)$, the non-abelian boson star obtained by deformation of the p -th BM solution also possesses $2p$ unstable modes. (Note also that we could not find non-trivial $p = 0$ solutions.) This is at least consistent with the stability of the boson star, i.e. for $p = 0$.

According to the arguments of the catastrophe theory, we have to look for bifurcations or the occurrence of cusps in the plot of e.g. the energy (or better the binding energy given by the difference $M - \alpha N$) as a function of the particle number N . Here the main difference between “pure” boson stars and the Yang-Mills boson stars resides in the fact that for the latter the mass of the solution does not tend to zero in the limit $N \rightarrow 0$ because it converges to the energy of the corresponding BM solution. As a consequence, the function $M - \alpha N$ is strictly positive for $N \ll 1$ suggesting that the binding energy is positive and conforing the statement that the $p > 1$ Yang-Mills boson stars are indeed unstable.

In the case $p = 1$ such a diagram indicates the occurrence of a cusp for the maximal value of the particle number, i.e. at $N \approx 0.35$, corresponding to $\phi(0) \approx 0.4$ (the results here correspond to $\alpha = 1$). No solution seems to exist for $N > 0.35$, however, the solutions existing for higher values of $\phi(0)$, i.e. $\phi(0) > 0.4$, form another branch with a cusp at $N \approx 0.35$. The binding energy of the second branch is higher than the one corresponding to the main branch. So it is likely that the solution constructed has two unstable modes for the first branch, i.e. for $\phi(0) < 0.4$ and three unstable modes for $\phi(0) > 0.4$. These results definitely need to be confirmed by a normal mode analysis of the linearized equations.

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References

- [1] D. J. Kaup, Phys. Rev. **172** (1968) 1332.
- [2] R. Ruffini and S. Bonazzola, Phys. Rev. **187** (1969) 1767.
- [3] E. W. Mielke and R. Scherzer, Phys. Rev. D **24** (1981) 2111.

- [4] M. Colpi, S. L. Shapiro and I. Wasserman, *Phys. Rev. Lett.* **57** (1986) 2485.
- [5] F. E. Schunck and D. F. Torres, *Int. J. Mod. Phys. D* **9** (2000) 601.
- [6] J. J. van der Bij and M. Gleiser, *Phys. Lett. B* **194** (1987) 482.
- [7] B. Gradwohl and G. Kaelbermann, *Nucl. Phys. B* **324** (1989) 215;
Z. J. Tao and X. Xue, *Phys. Rev. D* **45** (1992) 1878;
P. P. Fiziev, S. S. Yazadjiev, T. L. Boyadjiev and M. D. Todorov, *Phys. Rev. D* **61** (2000) 124018;
E. W. Mielke and F. E. Schunck, *Nucl. Phys. B* **564** (2000) 185;
E. W. Mielke and F. E. Schunck, *Gen. Rel. Grav.* **33** (2001) 805.
- [8] A. B. Henriques, A. R. Liddle and R. G. Moorhouse, *Nucl. Phys. B* **337** (1990) 737.
- [9] M. Gleiser, *Phys. Rev. D* **38** (1988) 2376 [Erratum-ibid. *D* **39** (1988) 1258].
- [10] M. Gleiser and R. Watkins, *Nucl. Phys. B* **319** (1989) 733.
- [11] T. D. Lee and Y. Pang, *Nucl. Phys. B* **315** (1989) 477.
- [12] F. V. Kusmartsev, E. W. Mielke and F. E. Schunck, *Phys. Rev. D* **43** (1991) 3895;
F. E. Schunck, F. V. Kusmartsev and E. W. Mielke, in *Approaches to Numerical Relativity*, ed. R. A. d’Inverno (Cambridge University Press: Cambridge 1992) pp. 130-140.
- [13] P. Jetzer, *Phys. Rept.* **220** (1992) 163.
- [14] E. W. Mielke and F. E. Schunck, in *Proc. of the Eighth Marcel Grossman Meeting on General Relativity*, ed. T. Piran and R. Ruffini (World Scientific: Singapore 1999), pp. 1607-1626.
- [15] F. E. Schunck and E. W. Mielke, *Class. Quant. Grav.* **20** (2003) R301.
- [16] P. Jetzer and J. J. van der Bij, *Phys. Lett. B* **227** (1989) 341.
- [17] R. Bartnik and J. McKinnon, *Phys. Rev. Lett.* **61** (1988) 141.
- [18] M. S. Volkov and D. V. Gal’tsov, *Phys. Rept.* **319** (1999) 1 [arXiv:hep-th/9810070].
- [19] F. R. Klinkhamer and N. S. Manton, *Phys. Rev. D* **30** (1984) 2212.
- [20] J. Kunz and Y. Brihaye, *Phys. Lett. B* **216** (1999) 343.
- [21] L. G. Yaffe, *Phys. Rev. D* **40** (1990) 3463.
- [22] B. R. Greene, S. D. Mathur and C. M. O’Neill *Phys. Rev. D* **47** (1993) 2242.
- [23] A. Prikas, *Phys. Rev. D* **70** (2004) 045008.
- [24] T. Akiba, H. Kikuchi and T. Yanagida, *Phys. Rev. D* **38** (1988) 1937.
- [25] I. Pena and D. Sudarsky, *Class. Quant. Grav.* **14** (1997) 3131.
- [26] F. V. Kusmartsev, *Phys. Rept.* **183** (1989) 1.
- [27] G. V. Lavrelashvili and D. Maison, *Phys. Lett. B* **343** (1995) 214; M. S. Volkov, O. Brodbeck, G. V. Lavrelashvili and N. Straumann, *Phys. Lett. B* **349** (1995) 438.