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Estimate of Cutting Tool Lifespan through Cox Proportional Hazards Model

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Abstract: An important proportion of the cost of machined mechanical elements emanates from the cost of cutting tools inserts. A large part of these costs is due to improper maintenance decisions. In this paper, we demonstrate the use of a gamma process tool wear simulation for providing tool lifetimes at several cutting speeds. We show the use of a survival model (Cox Proportional Hazards model) on turning cutting inserts, using the cutting speed as covariate. Contrary to other approaches, the Proportional Hazards model is not used to produce the parameters of a reliability model, but the Mean Up Time is rather computed through the integration of the raw survival model reliability baseline. Additional simulated data is used to validate the model. The importance of a proper distribution of the PH model fitting data is highlighted by the experimental results.

Keywords: Machining, Reliability, Lifetime, Survival analysis, Numerical simulation, Stochastic modelling, Degradation

1. INTRODUCTION

An important part of mechanical parts is associated with machining, of which up to 40% is only linked with tool management and improper use (Li (2012); Denkena et al. (2014)). The huge costs that result call for aids regarding tool maintenance decision-making processes. In this framework, and given the semi-continuous aspect of industrial machining, online Tool Condition Monitoring (TCM) is expected to provide real-time indication of the tool wear. It should help optimizing the lifetime of cutting tools while avoiding non-quality production, which would be even more deleterious than improper tool management, in particular in high added-value industries.

The crucial wear in machining is tool flank wear, as it has the most influence over surface roughness and dimensional accuracy. The online TCM can not rely on direct optical measurements of tool flank wear, therefore a range of tool wear indicator variables has to be considered. In particular, Abellan-Nebot and Subrión (2009); Siddhpura and Paurobally (2013); Rehorn et al. (2005) explored most possible indicators. Relevant bibliography generally holds vibratory, cutting forces and electrical power intake as relevant and easy to acquire.

Moreover, the evolution of tool flank wear is influenced by the machining parameters, i.e. (in turning) depth of cut, feed rate and cutting speed (Aramesh et al. (2015); Davim (2013)). This aspect is further discussed in section 2, which addresses the simulation of tool lifetimes.

Because of this influence of cutting parameters, one could consider the cutting parameters as explanatory variables to the tools lifetime, leading to perform survival analyses and models such as Cox Proportional Hazards (PH) model. This model, which was first introduced in Cox (1972) for survival rates in medical applications, has later been introduced in other fields that may benefit from survival analysis, including maintenance and machining (Aramesh et al. (2014)). The use of this model is further discussed in section 3.

In this paper, we first address the simulation of tool lifetimes and degradation in general (section 2). Then, based on this simulated data, we demonstrate the fit of a Cox PH model to cutting tools lifetime, with cutting speed as the explanatory variable in section 3. In the penultimate section, we discuss the results of this model fit and finally we conclude on the methodology for further uses of Cox model and survival analysis in predicting the Remaining Useful Life of cutting tools.

2. TOOL LIFETIME SIMULATION

Letot et al. (2016) recently developed a stochastic model modelling the degradation through a gamma process:

$$Z(t) = Z(t_0) + G(m(t), \lambda) \tag{1}$$

$Z(t_0)$ being the degradation value at initial time, $m(t)$ the shape function, λ the scale parameter and G the gamma process defined as follows:

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- G has independent increments; moreover, the increments are stationary if $m(t) = ct$ is linear in time
- $G(t_0) = 0$
- G is a continuous-time stochastic process and $\forall t_2, t_1$: $(t_2 \geq t_1 \geq 0)$, the increment $G(t_2) - G(t_1)$ follows a gamma distribution, with $m(t)$ its shape function and λ its scale parameter. Its density function is:

$$f(x) = \frac{\lambda^{m(t_2)-m(t_1)}}{\Gamma(m(t_2) - m(t_1))} x^{m(t_2)-m(t_1)-1} \exp(-\lambda m) \tag{2}$$

Γ being the gamma function defined as $\Gamma(m(t_2) - m(t_1)) = \int_0^{+\infty} x^{m(t_2)-m(t_1)-1} \exp(-x)dx$. The gamma function is defined along $\mathbb{R}^+$, and this implies that the stochastic process allows only positive increments, which matches the nature of degradations.

The expected value and variance of the gamma process are as in equations (3) and (4).

$$E(G(t)) = \frac{m(t)}{\lambda} \tag{3}$$

$$Var(G(t)) = \frac{m(t)}{\lambda^2} \tag{4}$$

Moreover, Letot et al. (2016) gives (from Nystad et al. (2012)) the shape function used for the three tool flank wear areas:

$$m(t) = c\,(\sinh(ab) + \sinh(a(t-b))) \tag{5}$$

a being the inflexion point localization parameter, b quantifying the derivative at the inflexion point and c being a proportionality parameter setting the size of the gamma process increments. Moreover, van Noortwijk (2009) states that the estimate of these parameters is achieved through a series of experimental observations of the cumulated degradation over time, either through maximum-likelihood or moments methods. In the maximum-likelihood case, the estimate is performed by maximizing the function given in eq. (6).

$$L(t_i, z_i|a, b, c, \lambda) = \prod_{i=1}^{n-1} \frac{\lambda^{m(t_{i+1}-m(t_i)}}{\Gamma(m(t_{i+1}-m(t_i))} (z_{i+1} - z_i)^{m(t_{i+1})-m(t_i)-1} \exp(-\lambda(z_{i+1} - z_i)) \tag{6}$$

However, due to the complexity of the shape function, this method may become problematic. The moments method on the other hand requires previous knowledge of the shape function $m(t)$ and the gamma process parameters a and b. This knowledge comes either from a regression or from expert judgement. From there on, the parameters $cand\lambda$ can be estimated from eq. (3) and (4). Letot et al. (2016) then provides the reliability associated with this gamma process, which is the distribution of the time at which the degradation crosses a given threshold z_c:

$$R^*(t|z_c, z_0^*, m(t), \lambda) = 1 - \frac{\int_{\lambda(z_c - z_0^*)}^{+\infty} x^{m(t)-1} \exp(-x)dx}{\int_0^{+\infty} x^{m(t)-1} \exp(-x)dx} \tag{7}$$

Because of the complexity of this analytical expression, Letot (2013) demonstrated the use of Monte-Carlo simulations in order to fit a reliability model from the degradation threshold crossing times. Letot (2013) also showed the computation of the Mean Residual Life (MRL) as:

$$\text{MRL}(t) = \frac{\int_t^{+\infty} R^*(y)dy}{R^*(t)} \tag{8}$$

The star indicating that the model is updated at each new observation z_0^*.

Because the choice has been made to base the model on the moments method, additional information is required in order to produce the expert judgement. Davim (2013) provides the usual tool life formula (Taylor's model):

$$v_c T^n = C_T \tag{9}$$

T being the tool life, n Taylor's exponent (obtained through a regression of experimental points), v_c the cutting speed, and C_T a constant that depends on the other cutting parameters. With the help of this formula, Letot et al. (2016) uses a first estimate for parameters a and b,

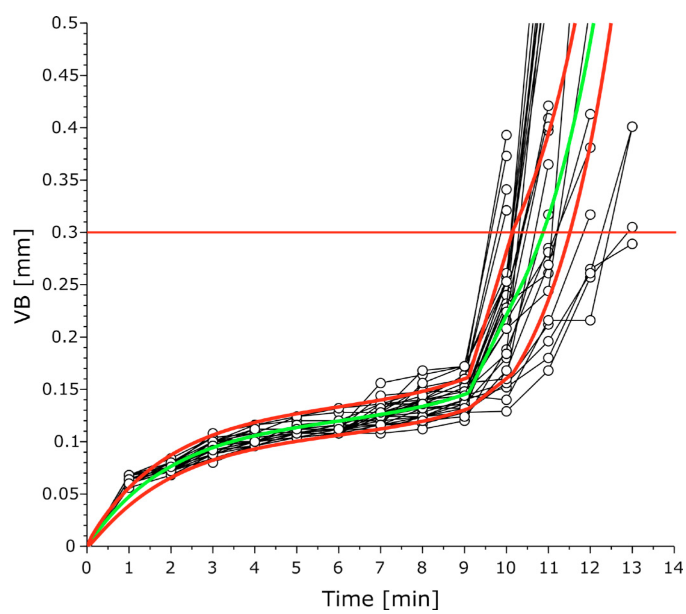

Fig. 1. Typical VB measurements on cutting tool inserts

then corrects them by a weighting factor based on v, v_{ref} the reference cutting speed (340 m/min) and n.

The model then allows the production of a large number of tools failure time (and degradation evolution) at several cutting speeds. The experimental data used for fitting this model originate from a CNC SOMAB "Unimab 450" turning center, using a DCLNL 2525M 12 cutting tool with SAFETY SA CNMG 1204 085B OR2500 tungsten carbide coated cutting inserts. Two of the three cutting parameters are set constant: depth of cut at 1.5 mm and feed rate at 0.18 mm/tr. These variable are to be considered constant for the remaining of this work. The material is a cylinder of gray cast iron with lamellar graphite FGL250 with a 322 Hv hardness. Its size is 220 mm long and 190 mm in diameter, and the VB_{max} is measured using a LEICA

MS5 optical microscope (uncertainty on the measurement: 2%).

The lifetime criterion used is $VB_{max} = 0.3$ mm in accordance with the ISO 3685 standard. Typical VB evolutions are displayed in fig. 1.

3. COX PH MODEL AND METHODOLOGY

Cox's PH Model is a common survival analysis model. It assumes that the failure rate is dependant both on the time and on covariates that describe the conditions under which the observations are made. In medical applications, a common covariate would be the age of the patient at the beginning of a treatment. In this particular case, the covariates should describe the cutting conditions. The PH model is expressed as follows:

$$h(t) = h_0(t) \cdot \exp\left(\sum_{i=1}^{p} \beta_i \alpha_i(t)\right) \qquad (10)$$

$h(t)$ being the failure rate, $h_0(t)$ the baseline hazard function, β_i the weighting factors for the p covariates and $\alpha_i(t)$ the covariates. Note that the model given in eq. 10 is the extended Cox model (Kleinbaum and Klein (2012)), involving time-dependant covariates $\alpha_i(t)$.

The Cox PH model baseline $h_0(t)$ and weighting factors β_i are computed with the help of the `survival` package for R. While this package would also allow a regression to a reliability model, such as Weibull, lognormal, exponential or gaussian, in the framework of this work, the raw baseline curve is kept unchanged. The intend is to avoid any bias or approximation (further than the Cox PH fit) in the reliability curve in order to assess the quality of Cox alone in the prediction.

In the framework of this study, the simulation tool will generate a large number of failure times for several cutting speeds. The covariate (there is only one in this case, i.e the cutting speed) is not intended to be time-dependant, reducing the $\alpha_i(t)$ to one constant value α. The unknown parameters therefore are $h_0(t)$ and β.

In order to assess the validity and the accuracy of this use of Cox's PH Model, the following experiments are performed:

(1) A first experiment constructs a presumed accurate PH model with 400 simulated lifetimes spread over 8 different cutting speeds. The quality of the model will then be assessed by 50 new lifetimes simulated at 10 cutting speeds of which some are different from the ones that were used to to construct the model and some are out of the bounds of the data that allowed to build the model.
(2) The second experiment fits a new PH with very limited data (i.e. 1 lifetime simulated at 5 different cutting speeds). At each iteration, given the next cutting speed, a MUT estimate is performed and compared with the observed lifetime. The observed lifetime is then added to the database and a new version of the PH model is computed. This experiment aims at showing the evolution of the error on MUT prediction as the statistical sample size increases.

	Cutting Speed [m/min]	Number of generated lifetimes
PH model fit	300	50
	320	50
	340	50
	360	50
	380	50
	400	50
	420	50
	440	50
Model validation	280	50
	300	50
	310	50
	320	50
	330	50
	340	50
	350	50
	410	50
	460	50

Table 1. Parameters for the PHM fit and validation runs

4. EXPERIMENTAL SETUP AND DISCUSSION

4.1 Experiment 1

The first experimental part aims at assessing the general validity of the PH model. Table 1 presents the cutting parameters and number of data points generated by the simulation model and used for the PH model fit and the validation. Let us remind that the other cutting parameters are constant values: the feed rate $V_f = 0.18$ mm/tr and the cutting depth $a_p = 1.5$ mm, and in general, the simulated data follows the experimental conditions provided in section 2.

In order to predict the lifetime of the cutting tool insert, we compute the MUT associated with the cutting insert. The MUT can be computed from the reliability function as follows:

$$MUT = \int_0^{+\infty} R(t)dt \qquad (11)$$

Because there is no supposed bias on the covariate, it is not normalized. The PH model however returns the failure rate baseline h_0 instead of a reliability. The survival (or reliability) function can be expressed as follows:

$$R(t) = exp(-\Lambda(t)) \qquad (12)$$

$\Lambda(t)$ being the cumulative hazard function. Therefore, the reliability function can be computed from the discrete baseline from Cox PH model h_0. Moreover, the survival analysis also provides the β relative to the cutting speed covariate. With $R_0(t)$ and β being now known, the PH model allows to compute an estimate of the MUT via eq. (11) for any cutting speed. The integration is performed through the trapezoidal rule.

However, as fig. 2 shows, the PH model is really accurate for $v_c \in [340, 440]$, despite the fitting points including cutting speeds as low as 300 m/min. An attempt to correct the behavior of the model at low v_c would be to increase the number of fitting points for the model at lower v_c. In a second phase of this experiment, low v_c points were added to the PH model database (see table 2). This addition of

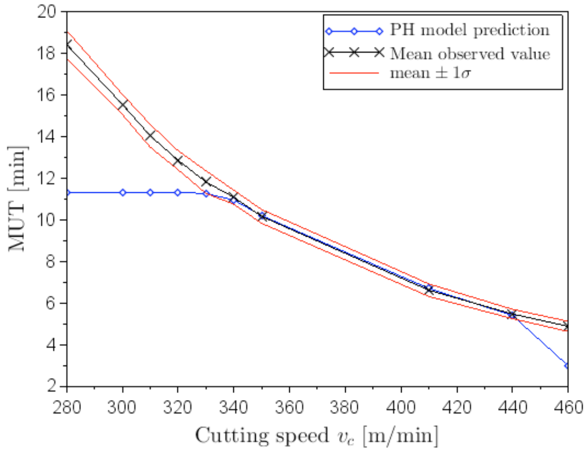

Fig. 2. Accuracy of the PHM prediction depending on the covariate value

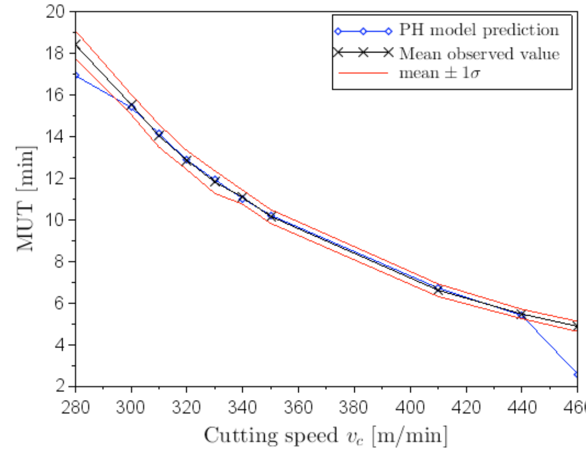

Fig. 3. Accuracy of the PHM prediction with extended data (including table 2)

	Cutting Speed [m/min]	Number of generated lifetimes
Additional PH model data	200	50
	220	50
	240	50
	260	50
	280	50

Table 2. Additional data for the PH fit

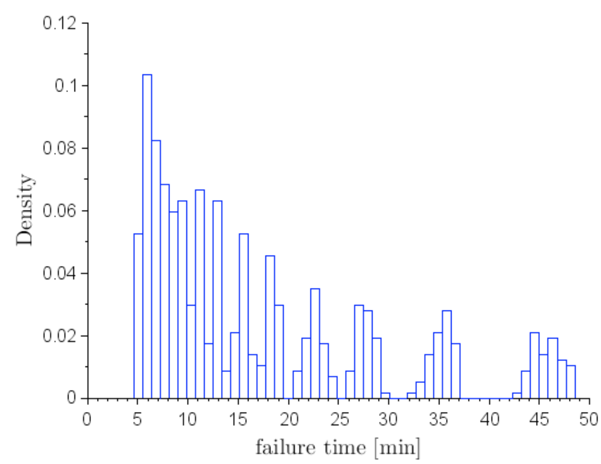

Fig. 4. Density analysis of the fitting sample per failure time (inluding tables 1 and 2)

data allowed to correct the curve until $v_c = 300$ m/min, as fig. 3 shows.

The cause of the model failure at low v_c is most likely the density of low failure times (i.e. high v_c) in the model. Because Cox's PH model fitting is performed through a maximum likelihood optimization, it is naturally lead to be more representative of high-density data than lower densities. An analogy for this effect would be the way that a linear regression is more influenced by a dense cloud of points over fewer dispersed data.

Considering fig. 4, which shows the density analysis of the sample used to fit the model (tables 1 and 2), the accuracy of the model at lower failure times (i.e. higher v_c) is explained by the important density of data in these areas. On the contrary, the lower density of data in higher failure time brings imprecision on that part of the model. This uneven density can be explained by the exponential nature of eq. (9): a sample evenly spread along the v_c axis will favor precision at high v_c and poor accuracy at low v_c. In order to circumvent this issue, one should aim for a more evenly-spread dispersion of data in the sample.

Experiment 1 shows that a Cox PH model can be fitted to survival data obtained from the simulation model described in section 2. Weaknesses in the fitting of this model are shown, and have been explained by the lack of data at low v_c for one part and especially by the poor spreading of the failure. Further experiments should be undertaken in order to assess this proposition.

4.2 Experiment 2

The objective of this part is to assess the increase of accuracy as the sample grows, and to identify a plausible minimal sample size for Cox fitting in practical experiments. In order to achieve this, a first sample of 5 data points (at $v_c = 240$, 300, 360, 400 and 440 m/min) is used to construct a first PH model. At this point, a MUT estimate is done for two given v_c (of which the final prediction is close to the observed reality). Then, the model is fed with data points taken in a random order from the initial data pool (tables 1 and 2).

Due to the little available data at this stage, the first 50 PH fits do not properly converge, although they allow to compute a first MUT estimate. Figures 5 and 6 show the evolution of the MUT convergence towards the mean observed value as the sample size increases for $v_c = 350$ m/min and $v_c = 410$ m/min (which both provide a correct MUT estimate in experiment 1). Generally speaking, these figures show that around fifty to sixty observations are necessary to obtain a first correct approximation of the actual

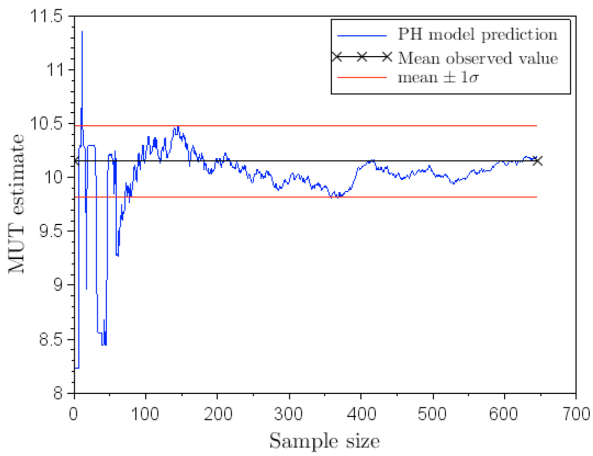

Fig. 5. Convergence analysis of MUT estimate at v_c = 350 m/min

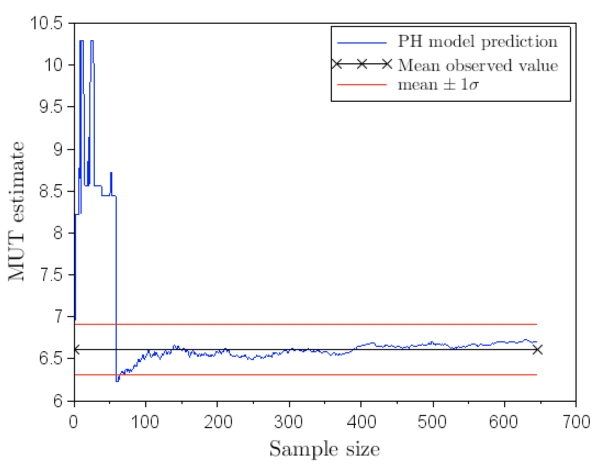

Fig. 6. Convergence analysis of MUT estimate at v_c = 410 m/min

observed value. With a perfect random input of data, this means that four to five observations of each v_c are sufficient to obtain the first approximate MUT. In the case of this study, a total of 120 observations (i.e. nine to ten observations of each point) provides a correct estimate.

In this framework, like in experiment 1, a new test with more evenly-spread failure times could prove valuable, as the convergence could be expected sooner, i.e. with less data points. However, this experiment shows that a satisfactory convergence can be achieved with limited data, and that the number of data points used to fit the PH model is large enough, although not spread enough.

5. CONCLUDING REMARKS

The experiments presented in this paper aimed at fitting a Cox PH model to data simulated through a stochastic model modelling the degradation through a gamma process. The experiments confirmed the quality of the model for portions of the cutting speed range. Lack of accuracy in some portions of the PH fit is thought to be related to the uneven spread of fitting data, although this proposition calls for further experiments.

During the experimental procedure, we also voluntarily used the experimental survival curve instead of fitting it to a reliability model. Because of the nature (maximum likelihood optimization) of the Cox fit, adding a second approximation during a regression to a reliability model does not seem inclined to provide better accuracy. Further experiments could also be performed in order to assess this hypothesis.

This work also calls for further experiments, including in particular multiple covariates and time-dependent covariates such as the ones online Tool Condition Monitoring should provide.

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