

# An overview of multiple criteria sorting methods

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## Examples of Sorting problems

Sorting = sorting into ordered categories

### Credit scoring

Assign applicants to categories labelled “accept”, “requires further examination”, “reject”, or referring to some other ordinal scale of risk evaluation, according to characteristics of the applicant.



### ASA score

Score assigned to patients by anesthesiologists before going to surgery depending on the patients medical parameters

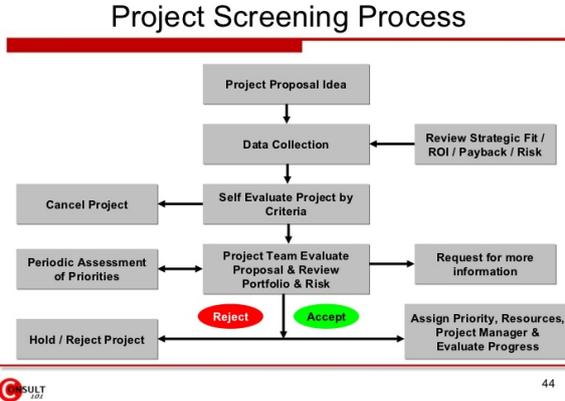
ASA Physical Status Classification	
1	Normal healthy person
2	Mild systemic disease that results in no functional limitation
3	Severe systemic disease that results in functional limitation
4	Severe systemic disease that causes a constant threat to life
5	Moribund patient not expected to live without the planned surgery
6	Brain-dead person whose organs are being removed for donation
E	Qualifier used for emergency procedures

Modified from American Society of Anesthesiologists: New classification of physical status. *Anesthesiology*. 1963;24:111.

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# Examples of Sorting problems (cont'd)

## Rating projects or candidates for a position



### A 3-STEP GUIDE TO THE CANDIDATE SCREENING PROCESS



# Examples of Sorting problems (cont'd)

## Grading students: pass or fail ; honors

PUPIL'S REPORT CARD																				
LAST NAME			FIRST NAME			AGE	GRADE	BUILDING												
Beason			Wleanor Jeanine			13	8	Roosevelt												
SCHOOL YEAR		Days	Miss	Third	Conduct	Spelling	Language	Arithmetic	Geography	History	Health	Music	Art	Physical						
1945-1946		Present	Adm.	Term	Effort	Book Lit.	Reading	Grammar	Writing	Phys. Ed.	Character	Attendance	Participation	Weight In						
PERIOD		Days	Miss	Third	Conduct	Spelling	Language	Arithmetic	Geography	History	Health	Music	Art	Physical						
1st Six Weeks		26	g		A	A	B	A	B	C	B	B	A	B	B	C	B			
2nd Six Weeks		17 1/2	14 1/2		A	A	C	B	B	B	B	B	C	A	A	C	C		67 1/2	98
3rd Six Weeks		24 1/2	3 1/2		A	A	A	A	B	A	B	B	A	A	A	C	B		62	108
Semester Grade		70	18		A	A	B	A	B	B	A	B	B	A	A	C	B			
4th Six Weeks		23			A	B	A	A	A	A	B	C	A	A	C	B		62 1/2	104 1/2	
5th Six Weeks		20 1/2	1 1/2		A	A	A	A	A	A	B	A	B	A	A	B	B		62 1/2	107
6th Six Weeks		31	0		A	A	B	A	B	B	B	A	A	A	C	B				
Semester Grade		88 1/2	1 1/2		A	A	B	A	B	B	B	B	A	A	C	B				
Yearly Grade		158 1/2	18 1/2		A	A	B	A	B	B	B	B	A	A	C	B				
Promoted to <u>HS</u> grade. Retained in <u>8</u> grade. Teacher <u>Florida Gardner</u>																				
EXPLANATION: Three "Honors" are awarded each semester, "Honors", "High Honors" and "Class Honors"																				

Course Grade	Interpretation	Grade Point Equivalent
A+*	Distinction	4.33
A		4.00
A-		3.67
B+	Good	3.33
B		3.00
B-		2.67
C+	Satisfactory	2.33
C		2.00
C-	Below Satisfactory	1.67
D**		1.00
F	Fail	0.00

# Characteristics of Sorting problems

These problems differ in several ways:

- ▶ assignments are to be made routinely or the decision problem is one-shot
- ▶ assignments are made by using explicit regulatory rules or they integrate the values or preferences of the decision maker(s)
- ▶ assignment data are available or not; they are abundant or not

In the sequel, we mainly focus

- ▶ on one-shot decisions
- ▶ the preferences of the DM have to be taken into account
- ▶ the number of assignment data that can be obtained is limited (a few dozens)

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## Multiple criteria sorting

- ▶ objects or alternatives sorted in ordered categories (preference)
- ▶ they are evaluated w.r.t. several criteria (direction of preference)
- ▶ assignment to categories respects *dominance* : if  $a$  is at least as good as  $b$  on all criteria, then  $a$  is not assigned to a worse category than  $b$

We assume w.l.o.g. that, for all criteria, the more the better (maximize evaluations)

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# Aim of this talk

Eyke Hüllermeier, at DA2PL 2014, Paris:

## MACHINE LEARNING VS. MCDA



	PL	MCDA	CA
<b>Models and model assumptions</b>	possibly weak assumptions (compensated by massive data)	stronger assumptions, axiomatic foundation	interpretable, often (generalized) linear models
<b>Model interpretation, usage, and expectations</b>	mainly predictive, accurate prediction of decision maker's behavior	mainly constructive or normative, convincing explanations of decisions	mainly descriptive, useful descriptions of decision makers
<b>Data availability</b>	data sets massively available (but not always accessible)	limited, user-generated data, no benchmark data	data abounds, many practical projects
<b>Data volume</b>	possibly very large („big data“)	typically small	moderate
<b>Validation, success criteria</b>	accuracy metrics, internal validation on data	user satisfaction (difficult to measure)	external evaluation (business oriented)
<b>Computational aspects</b>	scalability is critical	less critical (but short response time required)	less critical
<b>Application domains</b>	broad but typically not safety-critical (e-commerce, etc.), automated decisions	broad, possibly safety-critical, one-shot decisions	business and marketing

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## Aim of this talk (cont'd)

Leitmotiv : How can we validate the usage of a sorting method in MCDA ?

- ▶ Make a guided tour in the country of MC sorting methods
- ▶ Try to identify what has been done in order to validate the choice of a model and its usage
- ▶ Try to identify some issues that deserve further investigation

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## UTADIS

Utilité Additive DIScriminante (Jacquet-Lagrèze & Siskos 1982)

- ▶ Based on additive utility (additive value function)

$$u(a) = \sum_{i=1}^n u_i(a_i),$$

with  $u_i$  nondecreasing function of the evaluation  $a_i$  of  $a$  on criterion  $i$

- ▶ Categories  $C_1, \dots, C_h, \dots, C_H$ ; the greater  $h$  the better
- ▶ Thresholds  $U_h$

**Assignment rule:**

$$a \in C_h \quad \text{iff} \quad U_h \leq u(a) < U_{h+1}$$

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# ELECTRE TRI

Yu Wei PhD Thesis 1992, Roy & Bouyssou 1993

- ▶ Based on pairwise comparisons recorded in an *outranking relation*  $S$
- ▶ Principle:  $a$  outranks  $b$  ( $aSb$ ) is considered true if there are sufficient arguments to affirm that  $a$  is not worse than  $b$  and there is no essential argument to refuse this assertion
- ▶ Categories  $C_1, \dots, C_h, \dots, C_H$ ; the greater  $h$  the better
- ▶ For each category  $C_h$ , define a *lower profile*  $b^h$  of the category

**Assignment rule:** (pessimistic or pseudo-conjunctive)

$$a \in C_h \quad \text{iff} \quad aSb^h \text{ and } \text{Not}[aSb^{h+1}]$$

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## How to use such models for sorting ?

- ▶ Select a model (UTADIS or ELECTRE TRI or ...)
- ▶ Determine the model's parameters

Second issue first, using UTADIS as example.

- ▶ Direct elicitation
- ▶ Indirect elicitation or Learning

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# Determine the model's parameters

## Direct elicitation

- ▶ build an additive utility function by interacting with the DM methods: indifference judgments, SMART, direct rating, ...
- ▶ specify the minimal utility value  $U_h$  for category  $C_h$   
e.g., the DM specifies a (possibly artificial) alternative that has minimal utility in  $C_h$

## Indirect elicitation / Learning

Use assignment examples to infer the parameters of the model

Issues:

- ▶ indeterminacy (more than one model compatible with assignment examples)
- ▶ incompatibility (no model compatible with all assignment examples)

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## Indirect elicitation in MCDA

- ▶ Often the case that the number of assignment examples is not sufficient to determine a model with reasonable precision → indeterminacy / no incompatibility
- ▶ In case the DM is available for answering assignment questions, incremental elicitation strategies can be envisaged (cf. active learning)
- ▶ In the sequel, we concentrate on the analysis of **indirect elicitation** in a static situation in which a **fixed and "small" set of assignment examples is available**

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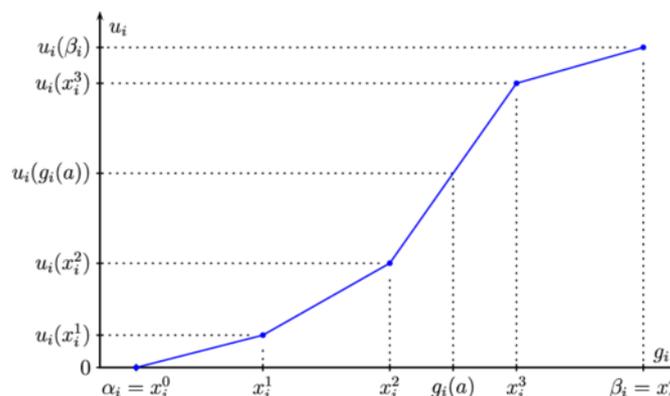
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## The case of UTADIS

- ▶ Learning method known as *preference disaggregation* or *ordinal regression*
- ▶ Resolution of a Linear Programming formulation (Jacquet-Lagrèze & Siskos 1982)

### Piecewise linear marginal utilities

$u(a) = \sum_{i=1}^n u_i(a_i)$  with  $u_i$  piecewise linear (number of pieces is a parameter)



$$g_i(a) = a_i ; \quad \sum_i u_i(\beta_i) = 1 ; \quad u_i(x_i^j) = u_i^j$$

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## A linear programming formulation

$\mathcal{A}_*$  = set of assignment examples =  $\bigcup_h \mathcal{A}_*^h$

$\mathcal{A}_*^h$  = alternatives assigned to  $C_h$

### Linear program (LP1)

$$\begin{aligned} \max \quad & \delta \\ \text{s.t.} \quad & u(a) \geq U_h + \delta \quad \forall a \in \mathcal{A}_*^h, \forall h \\ & u(a) \leq U_{h+1} - \epsilon - \delta \quad \forall a \in \mathcal{A}_*^h, \forall h \end{aligned}$$

Variables : thresholds  $U_h$  ; utilities at breakpoints  $u_i^j$  ;  $\delta$

### Comments

- ▶ If the LP has a solution, the latter assigns the examples a utility that is as far as possible from the thresholds  $U_h$
- ▶ If the LP has no solution, there are other LP formulations that are tolerant to “assignment errors”

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## Formulation minimizing constraint violations

### Linear program tolerant to “errors” (LP2)

$$\begin{aligned} \min \quad & \sum_{a \in \mathcal{A}_*} \sigma(a) \\ \text{s.t.} \quad & u(a) + \sigma(a) \geq U_h \quad \forall a \in \mathcal{A}_*^h, \forall h \\ & u(a) - \sigma(a) \leq U_{h+1} - \epsilon \quad \forall a \in \mathcal{A}_*^h, \forall h \end{aligned}$$

Variables : thresholds  $U_h$  ; utilities at breakpoints  $u_i^j$  ;  
 $\sigma(a), a \in \mathcal{A}_*$

### Comments

- ▶ Such LP can deal with large sets of assignment examples
- ▶ The solution is not necessarily optimal in terms of accuracy; no error model is postulated here

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## Solution selection : centrality

Assume (LP1) has a solution

This solution maximizes the minimal utility difference between examples belonging to consecutive categories

A major idea for selecting a solution : centrality

### Seeking for centrality

**UTADIS\*** computes the average of  $2n$  *extreme* solutions

**ACUTA** (Bous et al. 2010) computes the *analytic centre* of the polyhedron defined by the constraints in parameters space

**Chebyshev centre** (Doumpos et al. 2014) selected solution is the centre of the larger sphere inscribed in the feasible polyhedron

Interesting experimental comparison of all these and other variants in Doumpos et al. 2014

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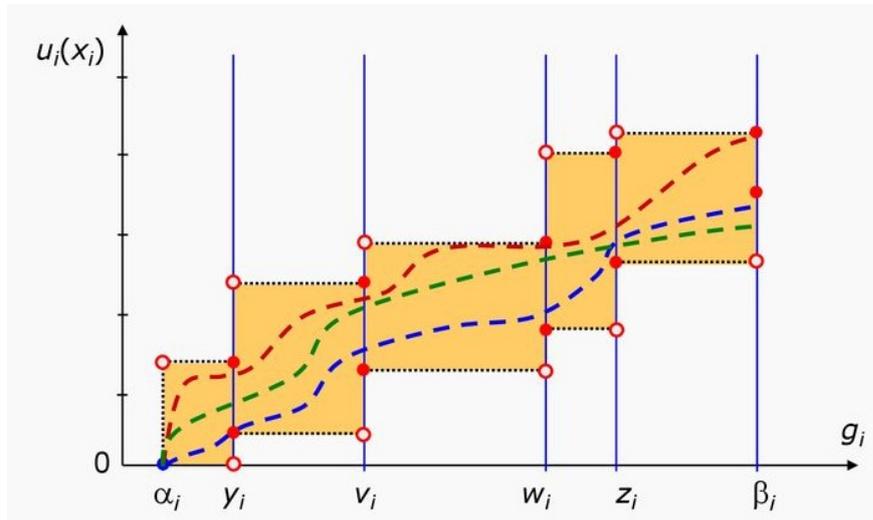
## Solution selection : most representative value function

- ▶ Greco et al. 2011
- ▶ Completely different idea, not related to centrality
- ▶ Emerged in the framework of ROR (Robust Ordinal Regression), Greco et al. 2008

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## Robust ordinal regression

- ▶ UTADIS<sup>GMS</sup> (Greco et al. 2010)
- ▶ Working with *all* additive utility functions compatible with the assignment examples
- ▶ Not only piecewise linear marginal utilities

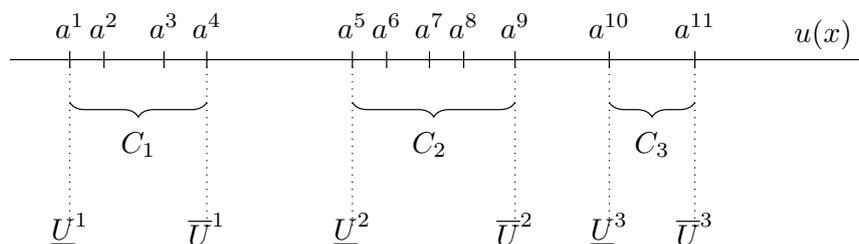


Variables =  $u_i^j = u_i(a_i^j)$  for all  $a_i^j \in \mathcal{A}_*$  ( $a_i^j$  denoted  $y_i, v_i, w_i, z_i$  in Figure); no category thresholds

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## Assignment rule in ROR

For any compatible utility function  $u$



- ▶ If  $u(a)$  falls in interval  $[\underline{U}^1, \overline{U}^1]$ ,  $a$  is assigned to  $C_1$
- ▶ If  $u(a)$  falls between  $\overline{U}^1$  and  $\underline{U}^2$ ,  $a$  is assigned to  $C_1$  or  $C_2$
- ▶ ...

For each compatible utility function, each alternative is assigned to one category or an interval of two consecutive categories

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# Possible and necessary assignments

## Robust assignment rules

- ▶  $C_h$  is a *possible* assignment for  $a$  if **there is** a compatible utility function  $u$  assigning  $a$  to  $C_h$
- ▶  $C_h$  is a *necessary* assignment for  $a$  if **all** compatible utility functions assign  $a$  to  $C_h$

The possible and the necessary assignments of  $a$  are sets of consecutive categories. The set of necessary assignments may be empty.

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## Taking account of uncertainty, imprecision, indeterminacy

### The robustness idea

Many declinations to robust sorting, ranking, choosing:

Dias et al. 2002, GRIP (Figueira et al. 2008), Non-additive ROR (Angilella et al. 2010), ELECTRE<sup>GKMS</sup> (Greco et al. 2011), Hierarchy ROR (Corrente et al. 2012), UTADIS<sup>GMS</sup>-Group (Greco et al. 2012), UTADIS<sup>GMS</sup>-INT (Greco et al. 2014)

### Stochastic approach : SMAA

- ▶ SMAA-TRI (Tervonen et al. 2009): studies the sensitivity of assignment when the parameters of an ELECTRE TRI model are considered uncertain (probability distribution assumed on parameters space)
- ▶ For each alternative  $a$  and category  $C_h$ , it computes, by simulation, a *category acceptability index* = share of possible parameters values leading to assigning  $a$  to  $C_h$
- ▶ Kadzinski and Tervonen 2013 combine ROR and SMAA in case of indirect elicitation of ELECTRE TRI

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## Comments about the ROR approach

1. ROR aims to use the assignment examples without adding arbitrary information at the price of increasing imprecision in the assignments

Kadzinski and Tervonen 2013: *“Our experiences indicate that the range of possible assignments can be rather wide, whereas the set of necessary assignments is often empty”*

2. Ends up with a family of not explicitly identified models  
Ending up with a single hypothetical model describing the assignment mechanism has a value in terms of explainability

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## Comments about the ROR approach (cont'd)

3. In case the goal of identifying an hypothetical assessment model is abandoned, what is the justification of working with a particular parameterized family of models (such as additive utility or ELECTRE TRI or ...)?

Why not using the most general family of models, i.e., the monotone assignment models?

In any case, observe that the possible and necessary assignments depend on the definition of the parameterized family of models

The latter comment drives us to the model selection issue

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## Why working with a particular type of model ?

- ▶ In Machine learning / Preference learning, prediction accuracy is a major criterion
- ▶ In MCDA, the model has to appeal to (“speak” to) the DM and make sense for the *analyst* (= expert in MC methods)

### The model as a communication and reasoning tool

A model has

- ▶ an intuitive content
- ▶ and a formal or technical content implementing or operationalizing its intuitive content

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## Intuitive content of models (for ranking, choosing, sorting)

**Additive utility** sophisticates *weighted sum* (Keeney Raiffa 1976)

**Outranking relations** based on pairwise comparisons,  
independently of the other alternatives (Roy 1968)

- ▶ comparing  $a$  and  $b$  : balancing pros and cons
- ▶ different operationalizations of this “balance of pros and cons” : ELECTRE (Roy Bouyssou 1995), PROMETHEE (Brans Vincke 1985)

**AHP** builds a value function through pairwise comparisons organized in a hierarchy (Saaty 1980)

**TOPSIS** minimizes distance to ideal and maximizes distance to anti-ideal (Hwang Yoon 1981); many variants

**Methods based on the idea of clustering** Define class centers and assign to the “closest” center

- ▶  $k$ -means, ELECTRE TRI-C, Flowsort, Chen et al. 2007, 2008, ...
- ▶ Do not always guarantee monotonicity

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## Intuitive content of models (cont'd)

- ▶ Lots of ideas that can constitute a basis for communicating with the DM
- ▶ These intuitive ideas are implemented in, sometimes, complicated models and methods

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# Example of implementation: ELECTRE TRI

- ▶ For simplicity: sorting the set of all alternatives  $X$  in 2 categories  $\mathcal{A}$  (acceptable) and  $\mathcal{U}$  (unacceptable)
- ▶ Lower limit profile of category  $\mathcal{A}$ :  $p = (p_1, \dots, p_n)$
- ▶ Assignment rule (pessimistic or pseudo-conjunctive)

$$a \in \mathcal{A} \quad \text{iff} \quad aSp$$

## Outranking relation $S$

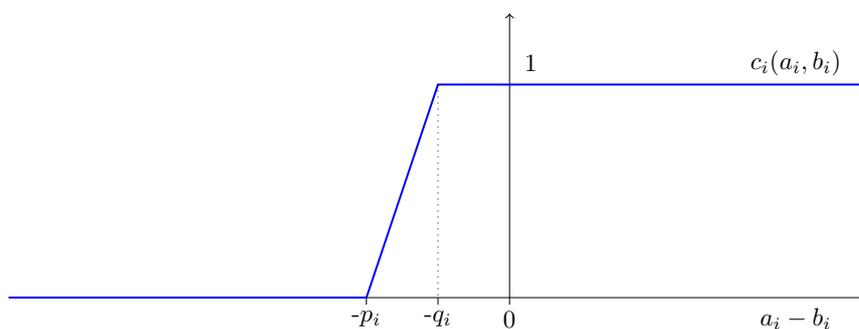
Defined for all  $a, b \in X$ . We have  $aSb$  if

- ▶  $a$  is “at least as good” as  $b$  w.r.t. a sufficient subset of criteria (measured by a *concordance* index)
- ▶ and there is no criterion on which  $a$  is unacceptably worse than  $b$  (measured by a *discordance* index)

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## Concordance

Marginal concordance index  $c_i(a_i, b_i)$



$q_i$  : indifference threshold

$p_i$  : preference threshold

gradual concordance between  $-p_i$  and  $-q_i$

Concordance index  $c(a, b)$

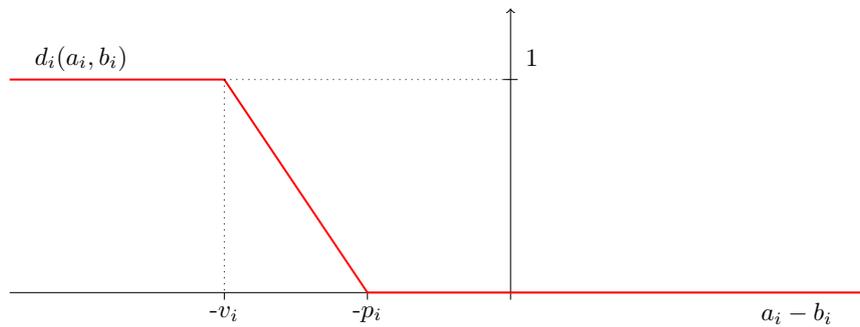
$$c(a, b) = \sum_{i=1}^n w_i c_i(a_i, b_i)$$

$w_i$  : weight of criterion  $i$

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# Discordance

## Marginal discordance index $d_i(a_i, b_i)$



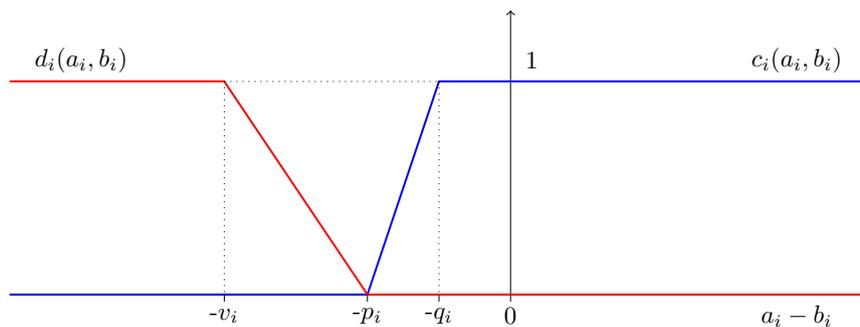
$v_i$  : veto threshold

$p_i$  : preference threshold

gradual discordance between  $-p_i$  and  $-q_i$

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## Outranking à la ELECTRE TRI



### Outranking relation $S$

$a$  outranks  $b$ , i.e.,  $aSb$  if the *degree of credibility*

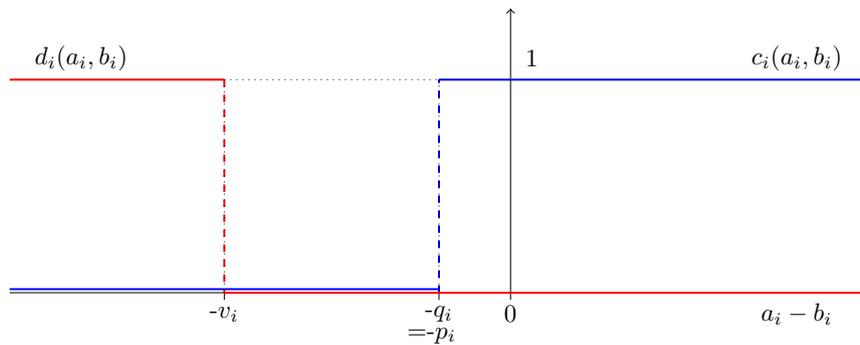
$$\sigma(a, b) = c(a, b) \prod_{i: d_i(a_i, b_i) > c(a, b)} \frac{1 - d_i(a_i, b_i)}{1 - c(a, b)}$$

passes some threshold  $\lambda$  with  $.5 \leq \lambda \leq 1$

Clearly, there are simpler ways of implementing the intuitive idea of outranking. This one favors a certain dose of graduality

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# Outranking à la ELECTRE I



## Outranking relation $S$

$a$  outranks  $b$ , i.e.,  $aSb$  if the *degree of credibility*

$$\sigma(a, b) = c(a, b) \prod_{i=1}^n (1 - d_i(a_i, b_i))$$

passes some threshold  $\lambda \in [.5, 1]$

In other words:  $aSb$  if  $c(a, b) \geq \lambda$  and  $d_i(a_i, b_i) = 0$  for all  $i$

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## Simpler ELECTRE TRI

The ELECTRE TRI model using the outranking relation à la ELECTRE I has inspired

- ▶ the Non Compensatory Sorting model (NCS model) characterized by Bouyssou Marchant 2007
- ▶ the Majority Rule sorting model (MR-Sort, Leroy et al. 2011, Sobrie et al. 2019) and the Majority Rule sorting model with coalitional Veto (Sobrie et al. 2017),

Learning a MR-Sort model using a large set of assignment examples is computationally feasible (Sobrie et al. 2016, Sobrie et al. 2019)

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# Role of the analyst

## Responsibility of the analyst

Has to be convinced that the selected model and the way it is used

- ▶ can be rationally justified in the decision context
- ▶ are logically correct

To achieve these goals, need for theoretical analysis of the models and methods

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## Analysis of models and methods

- ▶ Manifesto (Bouyssou et al. 1993); Evaluation and decision models (Bouyssou et al. 2000, 2006)
- ▶ Analysis: study the properties and characterize models and methods

## Characterization

- ▶ as a procedure: e.g., ELECTRE TRI is the only procedure having properties A,B and C (in the spirit of characterizations of voting procedures in social choice)
- ▶ of the preferences that can be represented by a model

Helps to

- ▶ identify the key concepts underlying the models (e.g., tradeoffs, weights etc)
- ▶ and use them correctly in elicitation processes

In the rest: illustration of benefits of analysis

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## Monotone sorting models

### Setting

- ▶  $X_i$ , the scale of criterion  $i$ ;  $X_i \subseteq \mathbb{R}$ ;  $\geq$ , the natural order on  $X_i$
- ▶  $X = \prod_{i=1}^n X_i$ , the set of all possible evaluation vectors (= alternatives)
- ▶ partial order  $\geq$  on  $X$  :  $a \geq b$  if  $a_i \geq b_i, \forall i$
- ▶ assumption:  $X$  is finite
- ▶ the elements of  $X$  are sorted in 2 categories (for simplicity):  $\mathcal{A}$  (acceptable);  $\mathcal{U}$  (unacceptable)
- ▶ the partition  $\langle \mathcal{A}, \mathcal{U} \rangle$  of  $X$  is *monotone*: if  $b \in \mathcal{A}$  and  $a \geq b$  then  $a \in \mathcal{A}$ ; similarly for  $\mathcal{U}$

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## Result 1

### Characterization of monotone partitions

$\langle \mathcal{A}, \mathcal{U} \rangle$  is a monotone partition of  $X$

iff there is a nondecreasing function  $F(a_1, \dots, a_n)$  on  $X$  such that

$$a \in \mathcal{A} \quad \text{iff} \quad F(a_1, \dots, a_n) \geq 0$$

- ▶ Proof: immediate
- ▶ For a bit more sophisticated result, see: Goldstein 1991, Greco et al. 2001, Słowiński et al. 2002, Bouyssou Marchant 2007
- ▶ This is the decision-rule preference model underlying the Dominance-based Rough Set Approach (DRSA)

### Examples

- ▶ UTADIS:  $a \in \mathcal{A} \quad \text{iff} \quad F(a) = \sum_{i=1}^n u_i(a_i) - U_1 \geq 0$
- ▶ ELECTRE TRI:  $a \in \mathcal{A} \quad \text{iff} \quad F(a) = \sigma(a, p) - \lambda \geq 0$

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## Result 2

### Equivalent model

$\langle \mathcal{A}, \mathcal{U} \rangle$  is a monotone partition of  $X$

iff there is a set  $\{p^1, \dots, p^j, \dots, p^J\}$  of profiles (=alternatives), which do not dominate one another, and such that

$$a \in \mathcal{A} \quad \text{iff} \quad \exists j \text{ s.t. } a \geq p^j$$

- ▶ Proof:  $p^1, \dots, p^J$  are the *minimal elements* in  $\mathcal{A}$  w.r.t. the partial order  $\geq$
- ▶ Every monotone sorting method is equivalent to defining a set of minimal profiles and checking whether  $a$  dominates ( $\geq$ ) one of these profiles
- ▶ Every monotone sorting method can be viewed as a synthetic way of specifying a set of *minimally acceptable* elements

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## Example

### An ELECTRE TRI model

- ▶  $n = 3$
- ▶  $X_i \subset [0, 10]: X_i = \{0, .5, 1, 1.5, \dots, 9.5, 10\}$
- ▶ limit profile:  $p = (8, 7, 5)$
- ▶  $w_i = \frac{1}{3}, q_i = 1, p_i = 2, v_i = 4$
- ▶  $\lambda = .6$

There are 6 minimal elements in  $\mathcal{A}$  :

$(7,6,2), (7,4,4), (5,6,4), (7,5.5,3.5), (6.5,3,3.5), (6.5,5.5,4)$

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## Complexity of the model

### Minimal number of questions

- ▶ An oracle tells you that the DM follows the ELECTRE TRI model above when answering assignment questions
- ▶ oracle gives you all the model's parameters
- ▶ What is the minimum number of questions you have to ask the DM in order to make sure that the oracle is not cheating on you ?

### Answer

- ▶ Ask whether the 6 minimal elements are indeed acceptable
- ▶ and Ask whether the maximal elements in  $\mathcal{U}$  are indeed unacceptable

Since there are 10 maximal elements in  $\mathcal{U}$ , the minimal number of question to be asked is 16

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## Complexity depends on the granularity of evaluations scales

- ▶ Consider the same ELECTRE TRI model defined on the scales  $X'_j$  composed of all rationals with 1 decimal digit in  $[0, 10]$
- ▶ The number of minimal elements in  $\mathcal{A}$  is 92

### Conclusions

- ▶ The minimal number of questions needed to completely determine an ELECTRE TRI model quickly grows with the degree of precision (granularity) of the evaluations scales
- ▶ When few assignment examples are available, identifying an ELECTRE TRI model is illusionary. The robust approach will have to deal with a large number of compatible models
- ▶ Similar conclusions can be drawn when using UTADIS
- ▶ Note that the number of minimally acceptable alternatives with the *simpler* ELECTRE TRI model doesn't change with the granularity (3 in the example)

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## Discussion : complexity

In a learning situation with a small set of assignment examples:

- ▶ If the analyst is strongly convinced that a given model perfectly matches the decision situation, then use the model, regardless of its complexity. In case the model's complexity is high, using a robust approach is likely to produce very imprecise assignments
- ▶ Otherwise, use a simpler model. Or a family of nested models of increasing complexity. Start with the simplest model in the family. If there is a simple model compatible with the assignment examples, it's fine. Otherwise, increase the complexity until a feasible model is found

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## Discussion : choice of a family of models

- ▶ Why working with UTADIS rather than ELECTRE TRI, Flowsort, AHP Sort, TOPSIS-Sort or ... ?  
In spite of efforts made in terms of analysis of models, the choice of working with one of them seems to be mainly governed by habits, education and practice
- ▶ Why not going more radically robust by eliciting the parameters of two (or more) models belonging to different families? Then compare the assignment results and consider as robust those who are shared by the two approaches (Bouyssou et al. 2000, Chapter 6, Bisdorff et al. Chapter 8)

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## Research issues

- ▶ We observed that all monotone sorting methods are synthetic ways of defining more or less complex sets of minimally acceptable alternatives.  
→ Are there monotone sorting models that are reasonably complex, interpretable and easy to learn ?
- ▶ Is it possible to approximate the assignment behavior of, say, ELECTRE TRI by a less complex model? Same question for UTADIS and MR-Sort
- ▶ For the record : study the motivations for working with a particular family of models

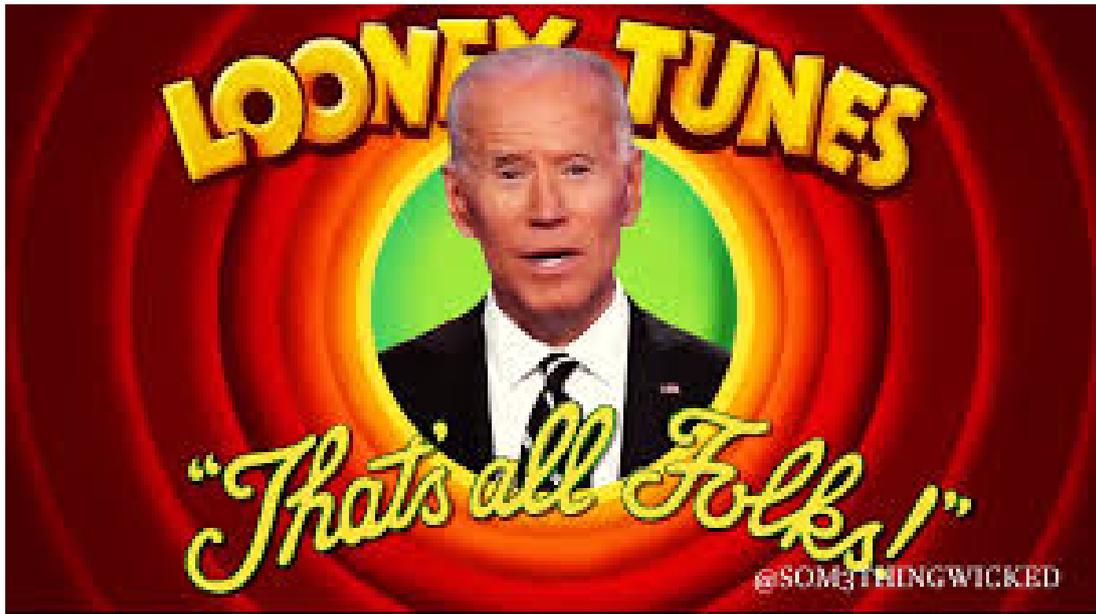
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Thank you for attention!



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Or else ...



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