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STABILITY OF ADS BLACK STRINGS

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We review the recent developments in the stability problem and phase diagram for asymptotically locally *AdS* black strings. First, we quickly review the case of locally flat black string before turning to the case of locally *AdS* spacetimes.

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1. Introduction

This decade has witnessed a growing interest for solutions of general relativity in *AdS* spaces. This is due to the celebrated *AdS/CFT* correspondence conjecture¹, relating solutions of general relativity in asymptotically *AdS* spaces to conformal field theories defined on the conformal boundary of the *AdS* space. In this context, black hole solutions play an important role².

In more than four dimensional spacetime, the uniqueness theorem on black holes, guaranteeing that the horizon topology of a black object is always S^2 is no longer true. Various black objects have been constructed in higher dimensions, such as black strings with horizon topology $S^{d-3} \times S^1$ in contrast with black holes with horizon topology S^{d-2} .

On the other hand, in 1993, R. Gregory and R. Laflamme have shown that black strings and branes are unstable towards long wavelength perturbations³. The Gregory-Laflamme instability was originally discovered in the framework of asymptotically locally flat spacetimes but it is believed to be a generic feature of black extended objects. In particular, it will be argued that this instability persists in asymptotically locally *AdS* spacetimes, where a black string solution has been found recently by R. Mann, E. Radu and C. Stelea⁴.

This proceeding is organised as follows: we review the black string instability and phase diagram in asymptotically locally flat spacetimes in section 2 before turning to asymptotically locally *AdS* spacetimes in section 3.

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2. Asymptotically locally flat space

Throughout this section, we consider the d -dimensional Einstein-Hilbert action

$$S = \frac{1}{16\pi G} \int_{\mathcal{M}} \sqrt{-g} R d^d x + \frac{1}{8\pi G} \int_{\partial\mathcal{M}} \sqrt{-h} K d^{d-1} x, \quad (1)$$

where G is the d -dimensional Newton constant which we set to one, \mathcal{M} is the spacetime manifold, g is the determinant of the metric, R is the scalar curvature and K is the extrinsic curvature of the boundary manifold $\partial\mathcal{M}$. The equation of motion resulting from the variation of the Einstein-Hilbert action is given by

$$R_{MN} = 0, \quad M, N = 0, \dots, d-1, \quad (2)$$

where R_{MN} is the Ricci tensor.

The black string solution to equation (2) is given by

$$ds^2 = - \left(1 - \left(\frac{r_0}{r} \right)^{d-4} \right) dt^2 + \frac{dr^2}{\left(1 - \left(\frac{r_0}{r} \right)^{d-4} \right)} + r^2 d\Omega_{d-3}^2 + dz^2, \quad z \in [0, L], \quad (3)$$

where L is the length of the coordinate z , r_0 is the horizon radius and $d\Omega_{d-3}$ is the line element of the unit $(d-3)$ -sphere.

The black string (3) is characterised by thermodynamical quantities, namely the mass M , associated to the time translation, the tension \mathcal{T} , associated with z -translation invariance, the temperature T_H , which can be obtained by demanding regularity at the horizon in the euclidean section, the entropy S , defined as one quarter of the horizon area and the length in the extradimension L .

These thermodynamical quantities can be used to define a thermodynamical phase diagram in temperature-entropy coordinates $(T_H, S/L)$ or in a mass-tension diagram (μ, n) , with the dimensionless quantities $\mu = M/L^{d-3}$, $n = \mathcal{T}L/M^5$.

The uniform black string solution is subject to a dynamical instability which manifests itself already at the linearised level³. The equations for the perturbations admit unstable solutions with small wavenumber k in the extradirection as well as stable solutions with a large wavenumber. The wavelengths of the various modes are given by $\lambda = 2\pi/k$. There is a static solution between these two regimes for $k = k_c$, where k_c is called the critical wavenumber.

This dynamical instability is related to the thermodynamical stability of the black string: long black strings are unstable while short black strings are stable. We refer the reader to the original paper³ for more details.

Initially, it was widely believed that the unstable black string should decay to an array of localised black holes but it has been shown that this decay would take an infinite proper time at the horizon⁶. This suggested the existence of another phase, the non uniform black string. Non uniform black strings were first constructed in a perturbative way then by solving numerically the full system of non-linear partial differential equations⁷. All these three phases, the black string, non uniform black string and the localised black hole are static solutions; these static solutions should be the equilibrium configurations since they don't evolve by definition.

A possible way to have an idea of the endpoint of the black string instability consists in comparing the thermodynamical properties of the three phases in a phase diagram⁵.

3. Asymptotically AdS space

In this section, we present the recent results obtained in the stability problem for black strings in AdS spacetime. We consider the d -dimensional Einstein-Hilbert action with a negative cosmological constant,

$$S = \frac{1}{16\pi G} \int_{\mathcal{M}} \sqrt{-g} \left(R + \frac{(d-1)(d-2)}{\ell^2} \right) d^d x + \frac{1}{8\pi G} \int_{\partial\mathcal{M}} \sqrt{-h} K d^{d-1} x, \quad (4)$$

with the same convention as in the previous section and where ℓ is the AdS radius, related to the cosmological constant Λ by $\Lambda = -(d-1)(d-2)/2\ell^2$.

The uniform black string solution is obtained by using the spherically symmetric ansatz

$$ds^2 = -b(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_{d-3}^2 + a(r)dz^2 \quad (5)$$

and solving numerically the equations of motion resulting from the variation of (4)⁴. The asymptotic behaviour of the metric fields is given by $a, b, f \approx r^2/\ell^2$ at the leading order.

The thermodynamical quantities characterising the solution (5) are the same as in the asymptotically locally flat case, but have to be computed in a regularised version of the action which diverges because of the AdS asymptotic. The regularised action is obtained by adding appropriate boundary counterterms⁸.

However, there is a new lengthscale in the theory, namely the AdS radius which affects the thermodynamical properties of the black string. In particular, it allows a new phase of thermodynamically stable black string, namely big AdS black strings, characterised by a large horizon radius r_0 - AdS length ratio. Small AdS black strings, with $r_0/\ell \ll 1$, are thermodynamically unstable.

We investigated the existence of a Gregory-Laflamme instability by considering non uniform black strings within the ansatz

$$ds^2 = -b(r)e^{2A(r,z)} + e^{2B(r,z)} \left(\frac{dr^2}{f(r)} + a(r)dz^2 \right) + e^{2C(r,z)} d\Omega_{d-3}^2, \quad z \in [0, L], \quad (6)$$

where a, b, f is the solution of Mann, Radu and Stelea and A, B, C are smooth functions of r and z . In the perturbative approach, we develop the non uniformity in a Fourier series of the variable z and in term of a small parameter ϵ according to $X(r, z) = \epsilon X_1(r) \cos(kz) + \epsilon^2 (X_0(r) + X_2(r) \cos(2kz)) + \mathcal{O}(\epsilon)^2$, X generically denoting A, B, C . Then we solve at each order in ϵ the corresponding equations of motion. Order ϵ^0 is just the uniform solution while order ϵ gives access to the linear stability problem. The modes X_1 correspond to the static perturbation in the Gregory-Laflamme picture. The equations for these modes form an eigenvalue problem, where

the eigenvalue is the square of the Gregory-Laflamme critical wavenumber. If the eigenvalue is real, there exists a Gregory-Laflamme instability, if it is imaginary, the solution is always stable⁹. It turns out that small *AdS* thermodynamically unstable black strings are dynamically unstable while big *AdS* thermodynamically stable black strings are dynamically stable, confirming the Gubser-Mitra conjecture¹¹ in this case.

The order ϵ^2 contains two independent modes: the backreactions, X_0 and massive modes X_2 . Thermodynamical corrections for the non uniform phase arise at this order where only the backreactions contribute¹⁰. We investigated the non uniform solutions emanating from the unstable modes at the order ϵ^2 for small *AdS* black strings. The new lengthscale provided by ℓ implies a new dimensionless quantity, $\mu_2 = L/\ell$ characterising the non uniform solutions. It turns out that for small value of μ_2 , the picture is essentially similar to the case of asymptotically locally flat spacetimes while for large μ_2 enough, the small *AdS* phase becomes thermodynamically stable. In other words, the length L plays an important role in the thermodynamical stability of non uniform black strings, just like r_0 does for uniform black strings; small ($r_0/\ell \ll 1$) and short ($L/\ell \ll 1$) *AdS* non uniform black string are thus thermodynamically unstable while small and long ($L/\ell \approx 1$) non uniform black strings are thermodynamically stable¹⁰.

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